

C4 INTEGRATION

Answers - Worksheet M

- 1 a**
- | | | | |
|--------------|---------|-----------|-----------|
| x | 1 | 3 | 5 |
| $x \ln(x+1)$ | $\ln 2$ | $3 \ln 4$ | $5 \ln 6$ |
- $\therefore \text{integral} \approx \frac{1}{2} \times 2 \times [\ln 2 + 5 \ln 6 + 2(3 \ln 4)] = 18.0$ (3sf)
- b**
- | | | | |
|----------|-----------------|----------------------|-----------------|
| x | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ |
| $\cot x$ | $\sqrt{3}$ | $\frac{1}{\sqrt{3}}$ | 0 |
- $\therefore \text{integral} \approx \frac{1}{2} \times \frac{\pi}{6} \times [\sqrt{3} + 0 + 2(\frac{1}{\sqrt{3}})] = 0.756$ (3sf)
- c**
- | | | | | | |
|----------------------|-------|-------|---|-------|-------|
| x | -2 | -1 | 0 | 1 | 2 |
| $e^{\frac{x^2}{10}}$ | 1.492 | 1.105 | 1 | 1.105 | 1.492 |
- $\therefore \text{integral} \approx \frac{1}{2} \times 1 \times [1.492 + 1.492 + 2(1.105 + 1 + 1.105)] = 4.70$ (3sf)
- d**
- | | | | | | |
|--------------------|-------|-------|-------|-------|-------|
| x | 0 | 0.25 | 0.5 | 0.75 | 1 |
| $\arccos(x^2 - 1)$ | 3.142 | 2.786 | 2.419 | 2.024 | 1.571 |
- $\therefore \text{integral} \approx \frac{1}{2} \times 0.25 \times [3.142 + 1.571 + 2(2.786 + 2.419 + 2.024)] = 2.40$ (3sf)
- e**
- | | | | | | | |
|------------------|--------|--------|--------|--------|--------|-----|
| x | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| $\sec^2(2x - 1)$ | 3.4255 | 2.0602 | 1.4680 | 1.1788 | 1.0411 | 1 |
- $\therefore \text{integral} \approx \frac{1}{2} \times 0.1 \times [3.4255 + 1 + 2(2.0602 + 1.4680 + 1.1788 + 1.0411)] = 0.796$ (3sf)
- f**
- | | | | | | | | |
|--------------|---|-------|-------|-------|-------|-------|-------|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $x^3 e^{-x}$ | 0 | 0.368 | 1.083 | 1.344 | 1.172 | 0.842 | 0.535 |
- $\therefore \text{integral} \approx \frac{1}{2} \times 1 \times [0 + 0.535 + 2(0.368 + 1.083 + 1.344 + 1.172 + 0.842)] = 5.08$ (3sf)
- 2 a**
- $$2 - \frac{1}{\sin x} = 0$$
- $$\sin x = \frac{1}{2}$$
- $$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$$
- $$x = \frac{\pi}{6}, \frac{5\pi}{6}$$
- b**
- | | | | | | |
|------------------------------|-----------------|-----------------|-----------------|------------------|------------------|
| x | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{5\pi}{6}$ |
| $2 - \operatorname{cosec} x$ | 0 | 0.8453 | 1 | 0.8453 | 0 |
- $\therefore \text{area} \approx \frac{1}{2} \times \frac{\pi}{6} \times [0 + 0 + 2(0.8453 + 1 + 0.8453)] = 1.41$ (3sf)
- 3 a**
- | | | | | |
|--------|----|--------|--------|--------|
| x | -1 | 0 | 1 | 2 |
| $f(x)$ | 0 | 0.5236 | 1.0472 | 2.0944 |
- $\therefore I \approx \frac{1}{2} \times 1 \times [0 + 2.0944 + 2(0.5236 + 1.0472)] = 2.62$ (3sf)
- b**
- | | | | | | | | |
|--------|----|--------|--------|--------|--------|--------|--------|
| x | -1 | -0.5 | 0 | 0.5 | 1 | 1.5 | 2 |
| $f(x)$ | 0 | 0.2709 | 0.5236 | 0.7763 | 1.0472 | 1.3717 | 2.0944 |
- $\therefore I \approx \frac{1}{2} \times 0.5 \times [0 + 2.0944 + 2(0.2709 + 0.5236 + 0.7763 + 1.0472 + 1.3717)] = 2.52$ (3sf)

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Answers - Worksheet M page 2

- 4 a**
- | | | | | | | | | | |
|---------|---|-----------|---------|-----------|---------|-----------|---------|-----------|---------|
| x | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
| $\ln x$ | 0 | $\ln 1.5$ | $\ln 2$ | $\ln 2.5$ | $\ln 3$ | $\ln 3.5$ | $\ln 4$ | $\ln 4.5$ | $\ln 5$ |
- i** $\approx \frac{1}{2} \times 2 \times [0 + \ln 5 + 2(\ln 3)] = 3.807$ (3dp)
- ii** $\approx \frac{1}{2} \times 1 \times [0 + \ln 5 + 2(\ln 2 + \ln 3 + \ln 4)] = 3.983$ (3dp)
- iii** $\approx \frac{1}{2} \times 0.5 \times [0 + \ln 5 + 2(\ln 1.5 + \ln 2 + \ln 2.5 + \ln 3 + \ln 3.5 + \ln 4 + \ln 4.5)] = 4.031$ (3dp)
- b** 2 \rightarrow 4 strips, increase = 0.176
 4 \rightarrow 8 strips, increase = 0.048
 e.g. suggest 8 \rightarrow 16 strips, increase \approx 0.013
 16 \rightarrow 32 strips, increase \approx 0.004
 32 \rightarrow 64 strips, increase \approx 0.001
 \therefore area $\approx 4.031 + 0.013 + 0.004 + 0.001 = 4.049$
- c** $u = \ln x, \frac{du}{dx} = \frac{1}{x}; \frac{dv}{dx} = 1, v = x$
- $$\int_1^5 \ln x \, dx = [x \ln x]_1^5 - \int_1^5 \frac{1}{x} \times x \, dx$$
- $$= [x \ln x - x]_1^5$$
- $$= (5 \ln 5 - 5) - (0 - 1)$$
- $$= 5 \ln 5 - 4$$
- $$= 4.047$$
- (3dp)

- 5** volume = $\pi \int_{-4}^0 (e^x - x)^2 \, dx$
- let $I = \int_{-4}^0 (e^x - x)^2 \, dx$
- | | | | | | |
|---------------|--------|-------|-------|-------|---|
| x | -4 | -3 | -2 | -1 | 0 |
| $(e^x - x)^2$ | 16.147 | 9.301 | 4.560 | 1.871 | 1 |
- $\therefore I \approx \frac{1}{2} \times 1 \times [16.147 + 1 + 2(9.301 + 4.560 + 1.871)] = 24.306$
- \therefore volume $\approx 24.306 \times \pi = 76.4$ (3sf)